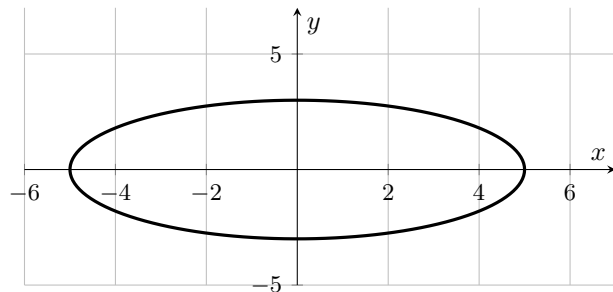


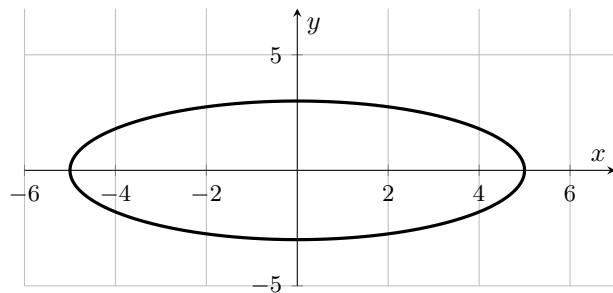
Question 5

Problem

A noncircular ellipse lacks an elementary perimeter formula. Explain why switching among Cartesian, parametric, and polar forms does not remove the essential difficulty.



Question 5 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. Each representation reduces to an elliptic integral.
4. Reparametrization changes appearance, not the intrinsic arc-length integral or its non-elementary nature.