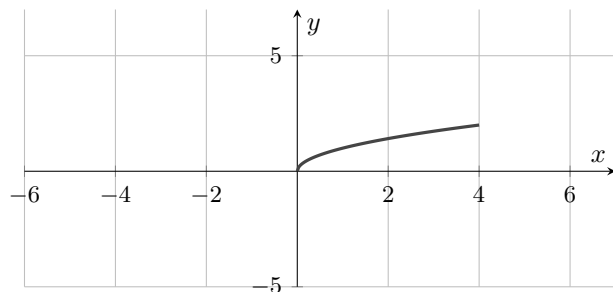


Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 4

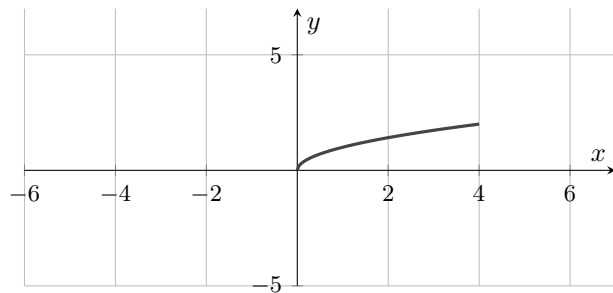
Problem

A surface is generated from $y = \sqrt{x}$, $0 \leq x \leq 4$, about the x -axis. Decide whether Cartesian or the parametrization $x = t^2, y = t$ is cleaner.



Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 4 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. The parametrization removes the singular derivative at $x = 0$:

$$ds = \sqrt{4t^2 + 1} dt, \quad S = 2\pi \int_0^2 t\sqrt{4t^2 + 1} dt.$$

Therefore the parametric form is cleaner.