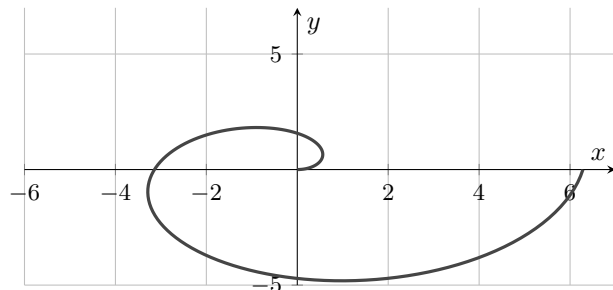


Question 3

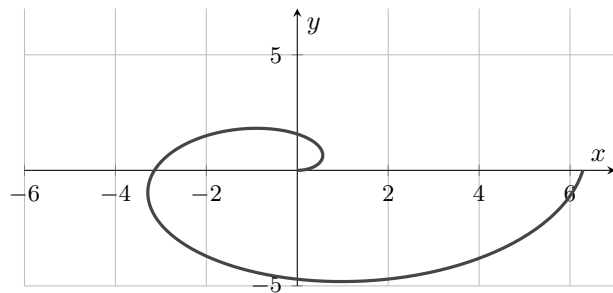
Problem

The spiral $r = \theta$ can be parametrized by $x = \theta \cos \theta, y = \theta \sin \theta$. Verify that both arc-length formulas give the same integrand.



Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 3 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. Polar gives $\sqrt{r^2 + (r')^2} = \sqrt{\theta^2 + 1}$.
4. Squaring the Cartesian derivatives cancels cross terms and gives the same $\sqrt{\theta^2 + 1}$.