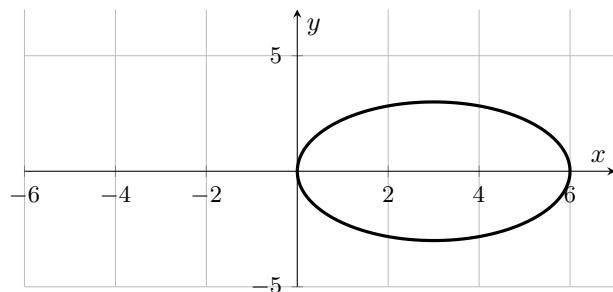


Calculus II Section 3-11: Arc Length and Surface Area Revisited

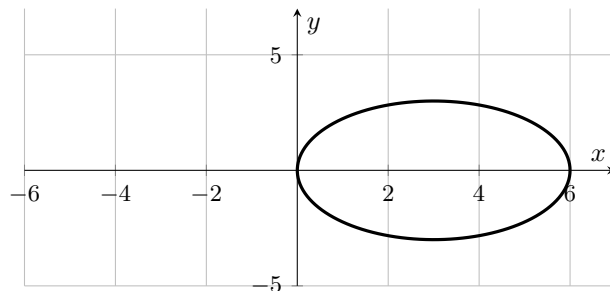
Question 2

Problem

A circle is given parametrically and polarly. Which representation makes its circumference easiest: $x = 3 + 3 \cos t$, $y = 3 \sin t$ or $r = 6 \cos \theta$? Explain.



Question 2 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. The parametric speed is 3 on $0 \leq t \leq 2\pi$. The polar arc-length factor is 6 on $-\pi/2 \leq \theta \leq \pi/2$. Each interval traces the circle once, so both forms give $L = 6\pi$.
4. The best choice is the one whose single-tracing interval is clearest.