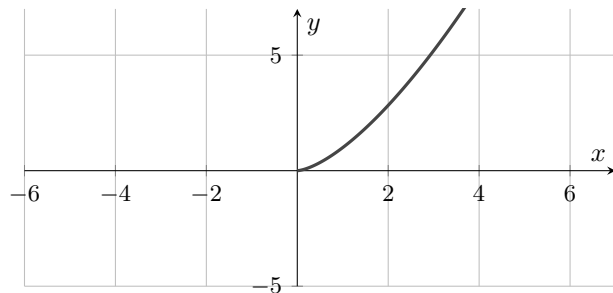


Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 1

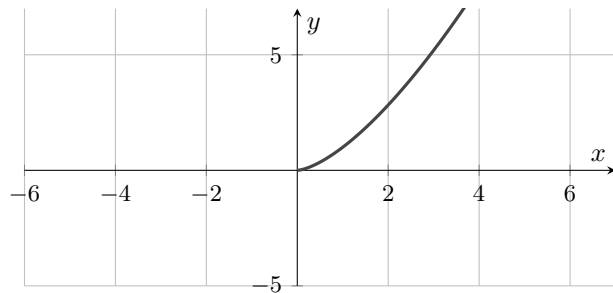
Problem

For $y = x^{3/2}$, $0 \leq x \leq 4$, compare the Cartesian and parametric arc-length setups using $x = t^2$, $y = t^3$.



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Question 1 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. Cartesian: $\int_0^4 \sqrt{1 + 9x/4} dx$.
4. Parametric with $0 \leq t \leq 2$: $\int_0^2 t\sqrt{4 + 9t^2} dt$.
5. Substitution $x = t^2$ makes them identical.