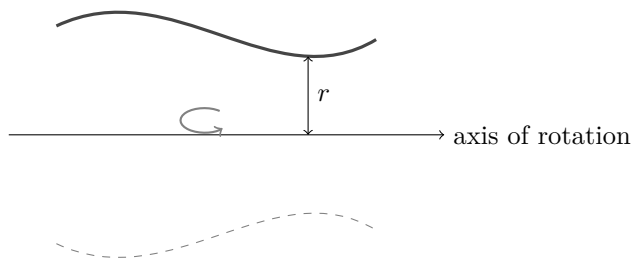


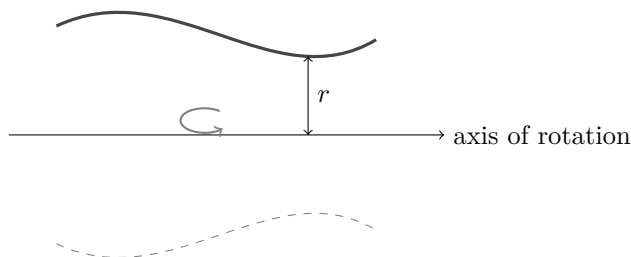
### Question 10

#### Problem

A student integrates a closed polar curve over  $0 \leq \theta \leq 2\pi$  and gets twice the expected surface area. Name two likely causes.



### Question 10 – Solution



### Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use  $|r \sin \theta|$  for the  $x$ -axis and  $|r \cos \theta|$  for the  $y$ -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. The curve may itself be traced twice on that interval, or two different portions of the generating curve may sweep the same surface after rotation.
4. Both are geometric double-counting.