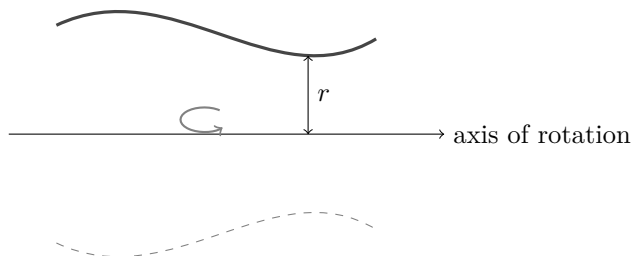


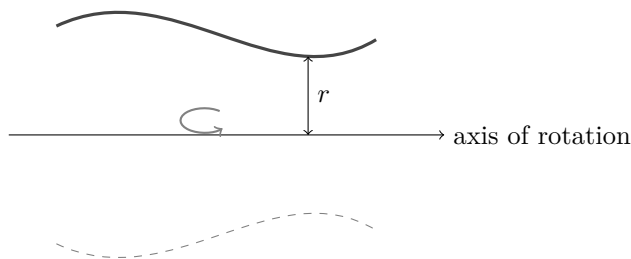
Question 9

Problem

Find the area generated when $r = 2 \cos \theta$, $0 \leq \theta \leq \pi/2$, rotates about the line $y = -1$.



Question 9 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. Here $ds = 2d\theta$ and radius is $y + 1 = 2 \sin \theta \cos \theta + 1$.

4. Hence $S = 4\pi \int_0^{\pi/2} (1 + 2 \sin \theta \cos \theta) d\theta = \boxed{2\pi^2 + 4\pi}$.