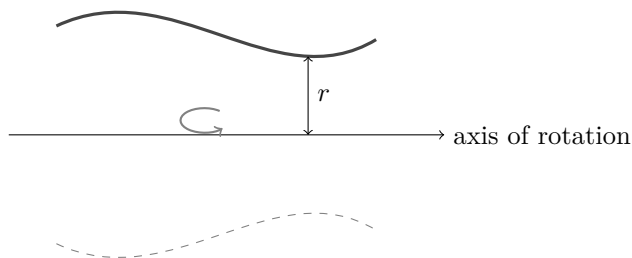


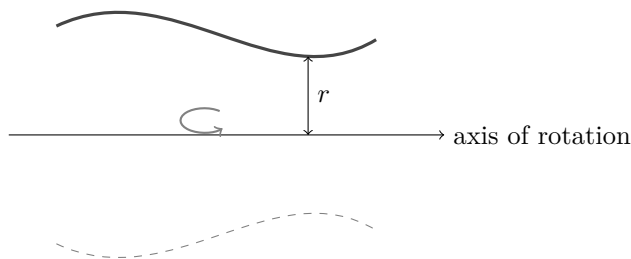
Question 8

Problem

A curve is entirely in the first quadrant. Compare its areas of revolution about the x - and y -axes using only weighted average coordinates.



Question 8 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. $S_x = 2\pi L \bar{y}_s$ and $S_y = 2\pi L \bar{x}_s$, where the bars are arc-length-weighted means.
4. Therefore $S_x/S_y = \bar{y}_s/\bar{x}_s$.