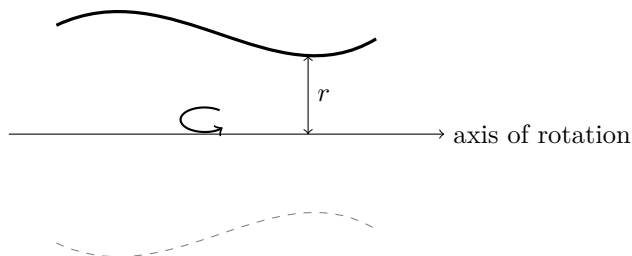


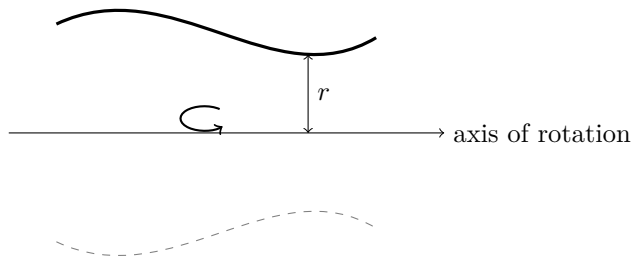
Question 7

Problem

Set up the surface area obtained by rotating one petal of $r = \cos 2\theta$, $-\pi/4 \leq \theta \leq \pi/4$, about the y -axis.



Question 7 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. Use radius $|x| = |r \cos \theta|$ and $ds = \sqrt{\cos^2 2\theta + 4 \sin^2 2\theta} d\theta$.
4. On this interval $r \cos \theta \geq 0$, so the requested setup is

$$S = 2\pi \int_{-\pi/4}^{\pi/4} \cos 2\theta \cos \theta \sqrt{\cos^2 2\theta + 4 \sin^2 2\theta} d\theta.$$