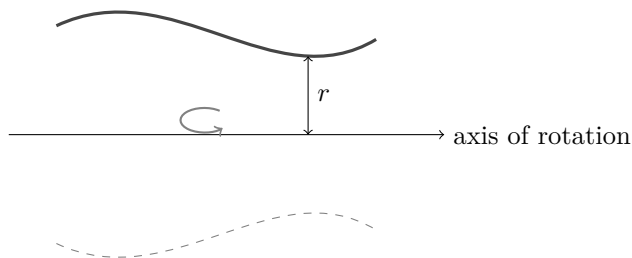


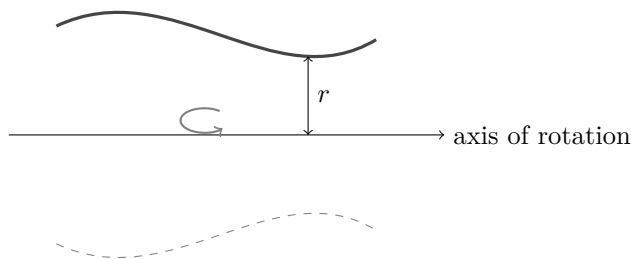
Question 6

Problem

For $r = R$, $0 \leq \theta \leq \pi$, rotate the semicircle about the x -axis and recover the sphere area.



Question 6 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. $y = R \sin \theta$ and $ds = R d\theta$.

4. Thus $S = 2\pi R^2 \int_0^\pi \sin \theta d\theta = \boxed{4\pi R^2}$.