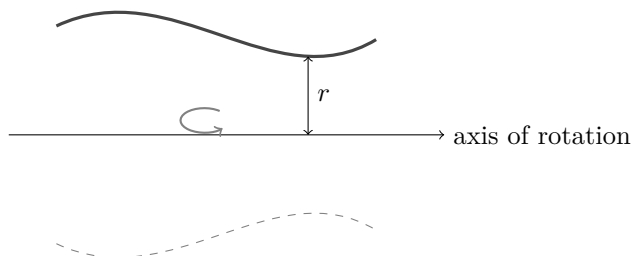


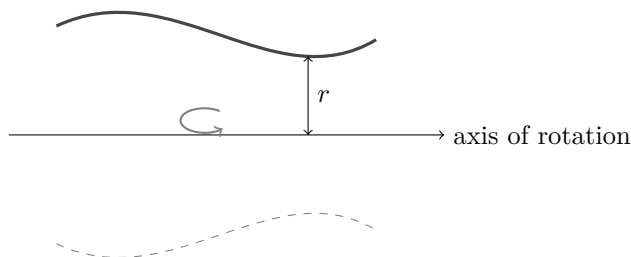
Question 5

Problem

A student proposes using only $0 \leq \theta \leq \pi/2$ when the circle $r = 4 \sin \theta$ rotates about the x -axis. Explain why that omits part of the surface and give a correct interval.



Question 5 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. The interval $0 \leq \theta \leq \pi/2$ traces only the right semicircle; rotation preserves the x -coordinate, so the left half is not duplicated.
4. Use $\boxed{0 \leq \theta \leq \pi}$ to trace the full generating circle once.