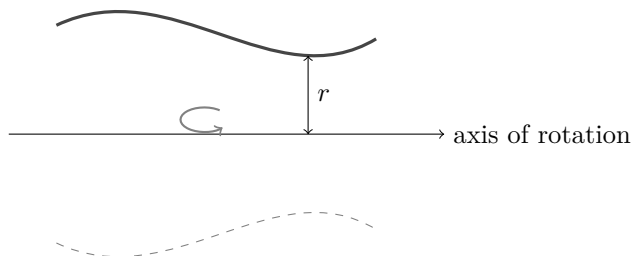


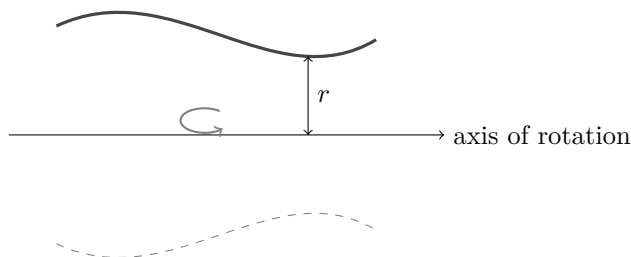
Question 4

Problem

Why must $|r \sin \theta|$ replace $r \sin \theta$ when a polar curve crosses the x -axis during rotation about that axis?



Question 4 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. Radius to the axis is the nonnegative distance $|y| = |r \sin \theta|$.
4. Without the absolute value, areas on opposite sides would cancel.