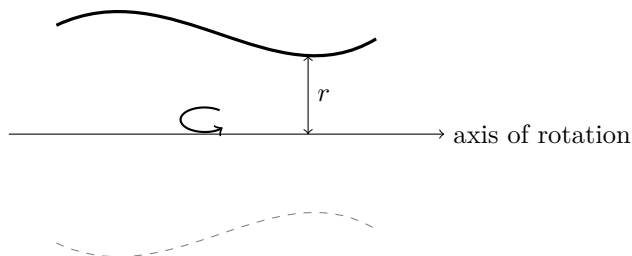


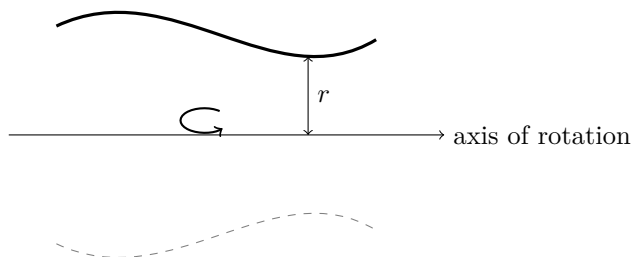
Question 3

Problem

Rotate the spiral arc $r = \theta$, $0 \leq \theta \leq \pi/2$, about the y -axis. Write the exact surface-area integral.



Question 3 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. Distance to the y -axis is $x = \theta \cos \theta$, and $ds = \sqrt{1 + \theta^2} d\theta$.

4. Thus
$$S = 2\pi \int_0^{\pi/2} \theta \cos \theta \sqrt{1 + \theta^2} d\theta.$$