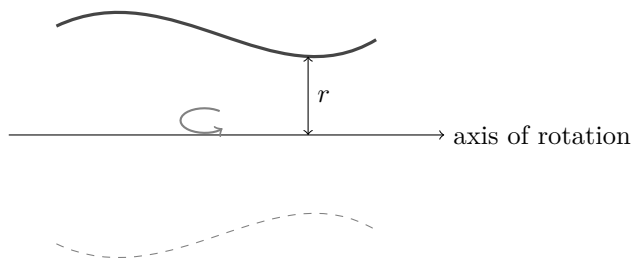


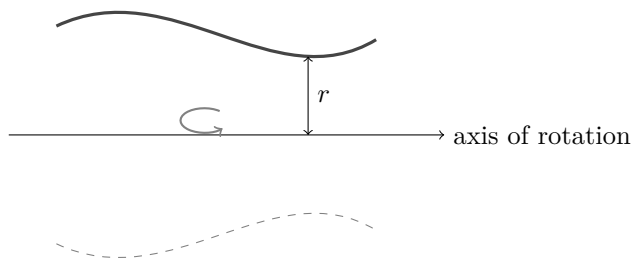
Question 2

Problem

Set up the surface area formed by rotating the cardioid $r = 1 + \cos \theta$, $0 \leq \theta \leq \pi$, about the polar axis.



Question 2 – Solution



Solution

1. Compute the polar arc-length element

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. Express the radius of rotation as a nonnegative distance: use $|r \sin \theta|$ for the x -axis and $|r \cos \theta|$ for the y -axis. Then apply

$$S = 2\pi \int_a^b (\text{radius to the axis}) ds,$$

over an interval that generates the surface exactly once.

3. Use $y = r \sin \theta$ and $ds = 2 \cos(\theta/2) d\theta$ on this interval.

4. Therefore

$$S = 4\pi \int_0^\pi (1 + \cos \theta) \sin \theta \cos(\theta/2) d\theta.$$