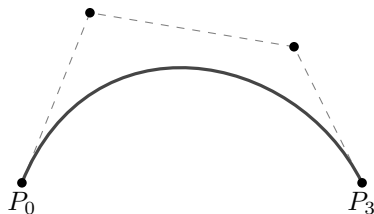
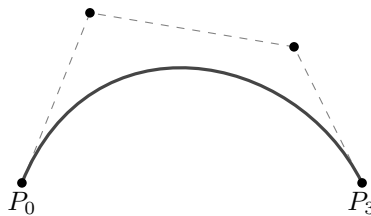


Question 10**Problem**

A cubic Bézier path is $\mathbf{r}(t) = (1-t)^3P_0 + 3(1-t)^2tP_1 + 3(1-t)t^2P_2 + t^3P_3$. Explain, without expanding, why it begins at P_0 and ends at P_3 .



Question 10 – Solution



Solution

1. The cubic Bézier path is

$$\mathbf{r}(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t) t^2 P_2 + t^3 P_3.$$

2. Evaluate the path at $t = 0$:

$$\begin{aligned}\mathbf{r}(0) &= (1-0)^3 P_0 + 3(1-0)^2 (0) P_1 \\ &\quad + 3(1-0)(0)^2 P_2 + (0)^3 P_3 \\ &= P_0.\end{aligned}$$

Every term containing a factor of t vanishes, leaving only P_0 .

3. Evaluate the path at $t = 1$:

$$\begin{aligned}\mathbf{r}(1) &= (1-1)^3 P_0 + 3(1-1)^2 (1) P_1 \\ &\quad + 3(1-1)(1)^2 P_2 + (1)^3 P_3 \\ &= P_3.\end{aligned}$$

Every term containing a factor of $1-t$ vanishes, leaving only P_3 .

4. Therefore,

$$\boxed{\mathbf{r}(0) = P_0 \quad \text{and} \quad \mathbf{r}(1) = P_3}.$$

The path begins at the first control point and ends at the last control point.