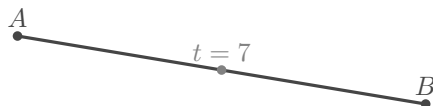
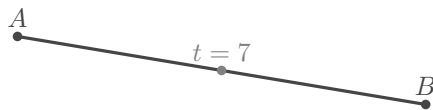


Question 6**Problem**

Find a parametrization of the line segment from $A = (-2, 5)$ to $B = (4, -1)$ that reaches its midpoint at $t = 7$, with $6 \leq t \leq 8$.



Question 6 – Solution



Solution

1. A line segment from A to B can be parametrized by

$$\mathbf{r} = A + u(B - A), \quad 0 \leq u \leq 1.$$

2. The parameter interval must be changed from $6 \leq t \leq 8$ to $0 \leq u \leq 1$. The linear change of variables is

$$u = \frac{t - 6}{8 - 6} = \frac{t - 6}{2}.$$

3. Compute the displacement vector:

$$B - A = (4, -1) - (-2, 5) = (6, -6).$$

Therefore,

$$(x, y) = (-2, 5) + \frac{t - 6}{2}(6, -6).$$

4. Simplify each coordinate:

$$x = -2 + 3(t - 6) = 3t - 20,$$

$$y = 5 - 3(t - 6) = 23 - 3t.$$

Hence a suitable parametrization is

$$\boxed{x = 3t - 20, \quad y = 23 - 3t, \quad 6 \leq t \leq 8}.$$

5. At $t = 6$ it gives $A = (-2, 5)$, and at $t = 8$ it gives $B = (4, -1)$. At $t = 7$ it gives

$$(x, y) = (1, 2) = \frac{A + B}{2},$$

so the midpoint condition is satisfied.