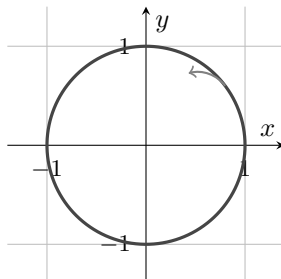
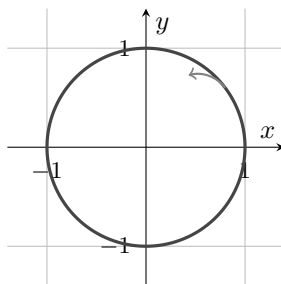


Question 5**Problem**

Two students parametrize the unit circle by $(\cos t, \sin t)$ and $(\cos 2t, -\sin 2t)$ on $0 \leq t \leq 2\pi$. Compare orientation and number of traversals.



Question 5 – Solution

**Solution**

1. For the first parametrization,

$$(x, y) = (\cos t, \sin t),$$

the polar angle is t . As t increases from 0 to 2π , the angle increases through one full revolution.

2. It begins at $(1, 0)$, then moves through $(0, 1)$, $(-1, 0)$, and $(0, -1)$ before returning to $(1, 0)$. Therefore, it traces the circle counterclockwise exactly once.

3. Rewrite the second parametrization as

$$(x, y) = (\cos(2t), -\sin(2t)) = (\cos(-2t), \sin(-2t)).$$

Its polar angle is therefore $-2t$.

4. As t increases from 0 to 2π , the angle decreases from 0 to -4π . A decreasing angle produces clockwise motion, and a total angular change of 4π represents two complete revolutions.

5. Thus

$(\cos t, \sin t)$ travels counterclockwise once,

whereas

$(\cos 2t, -\sin 2t)$ travels clockwise twice.