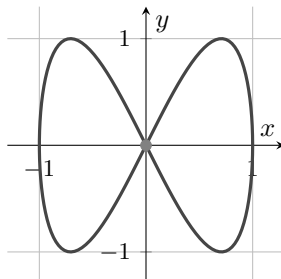
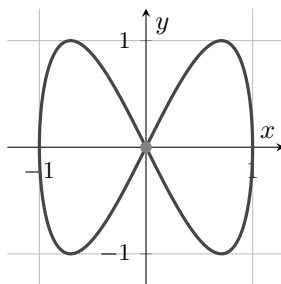


Question 4**Problem**

For $x = \cos t$, $y = \sin(2t)$, $0 \leq t \leq 2\pi$, locate the self-intersection and determine how many times it is visited.



Question 4 – Solution



Solution

1. A self-intersection occurs when two different parameter values produce the same point. The graph suggests checking the origin.
2. For the point to lie at the origin, first require $x = 0$:

$$\cos t = 0.$$

On $0 \leq t \leq 2\pi$, this gives

$$t = \frac{\pi}{2} \quad \text{or} \quad t = \frac{3\pi}{2}.$$

3. Check the y -coordinate at both values:

$$y\left(\frac{\pi}{2}\right) = \sin \pi = 0, \quad y\left(\frac{3\pi}{2}\right) = \sin 3\pi = 0.$$

Hence both parameter values produce $(0, 0)$.

4. The two visits correspond to different directions. Since

$$\frac{dy}{dx} = \frac{2 \cos(2t)}{-\sin t},$$

the slopes are 2 at $t = \pi/2$ and -2 at $t = 3\pi/2$.

5. Therefore, the curve crosses itself at

$$\boxed{(0, 0)},$$

and this point is visited exactly twice on the stated interval.