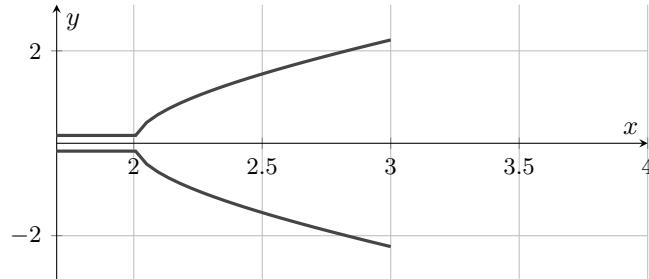


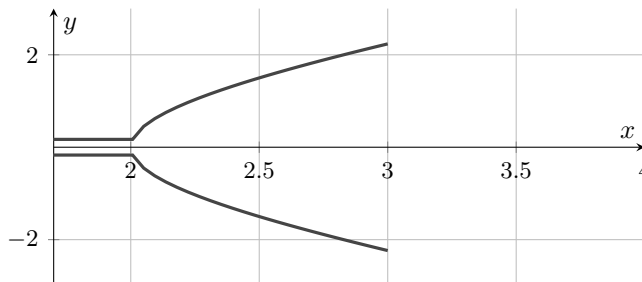
Question 1

Problem

A mechanism has $x = t + t^{-1}$ and $y = t - t^{-1}$, $t > 0$. Identify its Cartesian curve and explain which part the mechanism can reach.



Question 1 – Solution



Solution

1. Square both parametric equations:

$$x^2 = \left(t + \frac{1}{t}\right)^2 = t^2 + 2 + \frac{1}{t^2}, \quad y^2 = \left(t - \frac{1}{t}\right)^2 = t^2 - 2 + \frac{1}{t^2}.$$

2. Subtract the second equation from the first:

$$x^2 - y^2 = 4.$$

Thus the Cartesian curve is the hyperbola

$$\boxed{\frac{x^2}{4} - \frac{y^2}{4} = 1}.$$

3. Because $t > 0$, the arithmetic–geometric mean inequality gives

$$x = t + \frac{1}{t} \geq 2.$$

Therefore, the parametrization reaches only the right-hand branch of the hyperbola.

4. To determine the direction of travel, note that

$$\frac{dx}{dt} = 1 - \frac{1}{t^2}, \quad \frac{dy}{dt} = 1 + \frac{1}{t^2} > 0.$$

As t increases from 0 to 1, the point moves from the lower right toward $(2, 0)$. As t increases beyond 1, it moves from $(2, 0)$ toward the upper right.

5. Hence the mechanism traces the entire right branch exactly once, moving upward as t increases.