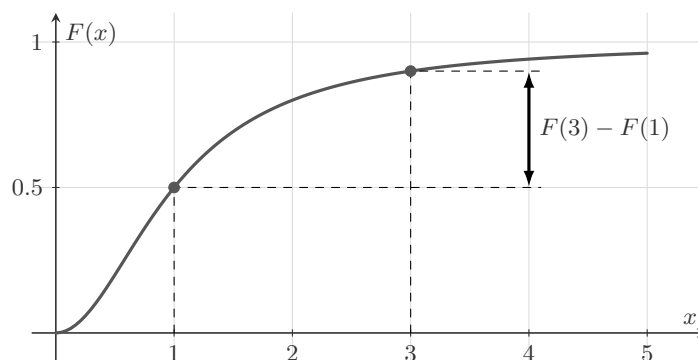
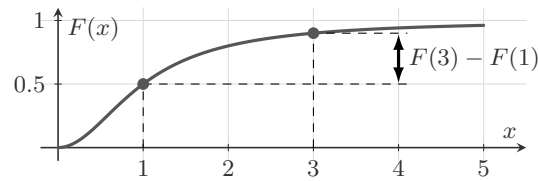


Question 10

A continuous random variable has CDF $F(x) = 0$ for $x < 0$, $F(x) = x^2/(1+x^2)$ for $x \geq 0$. Find the density and $P(1 < X < 3)$.



Question 10 – Solution



Step 1: Differentiate the CDF on each interval. For $x < 0$, $F(x) = 0$, so $f(x) = 0$. For $x > 0$,

$$\begin{aligned} f(x) &= \frac{d}{dx} \left(\frac{x^2}{1+x^2} \right) \\ &= \frac{2x(1+x^2) - x^2(2x)}{(1+x^2)^2} \\ &= \frac{2x}{(1+x^2)^2}. \end{aligned}$$

The CDF is continuous at $x = 0$, so there is no point mass there. Hence

$$f(x) = \begin{cases} 0, & x < 0, \\ \frac{2x}{(1+x^2)^2}, & x \geq 0. \end{cases}$$

Step 2: Use the CDF to compute the interval probability. For a continuous random variable,

$$P(1 < X < 3) = F(3) - F(1).$$

Now

$$F(3) = \frac{3^2}{1+3^2} = \frac{9}{10}, \quad F(1) = \frac{1^2}{1+1^2} = \frac{1}{2}.$$

Step 3: Subtract.

$$P(1 < X < 3) = \frac{9}{10} - \frac{1}{2} = \frac{9}{10} - \frac{5}{10} = \frac{4}{10} = \frac{2}{5}.$$

$$P(1 < X < 3) = \frac{2}{5}$$