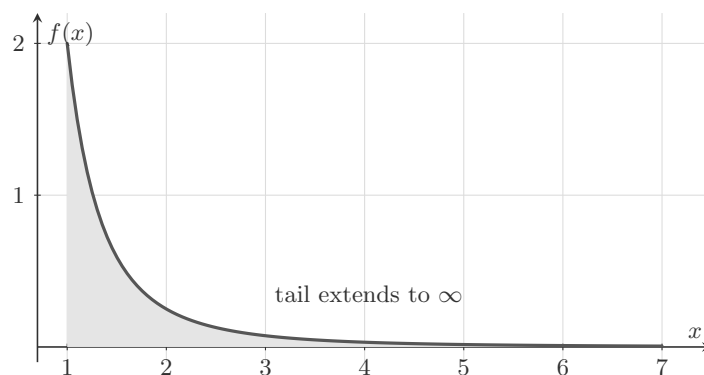
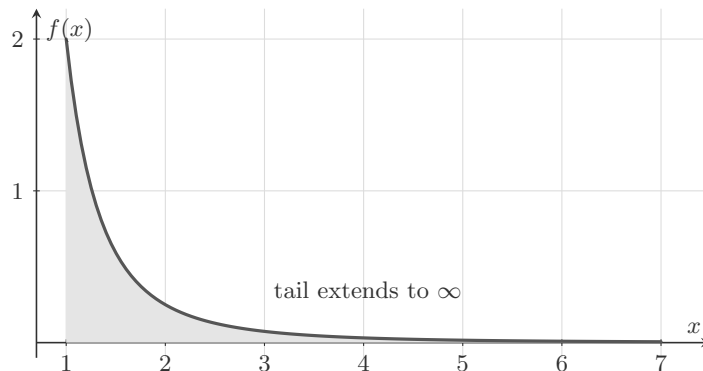


Question 9

Choose c so $f(x) = c/x^3$, $x \geq 1$, is a density. Determine whether $E[X]$ and $E[X^2]$ are finite.



Question 9 – Solution



Step 1: Normalize the density.

$$\begin{aligned} 1 &= c \int_1^{\infty} x^{-3} dx \\ &= c \left[-\frac{1}{2x^2} \right]_1^{\infty} = \frac{c}{2}. \end{aligned}$$

Thus

$$\boxed{c = 2}.$$

Step 2: Test the first moment.

$$\begin{aligned} E[X] &= \int_1^{\infty} x \frac{2}{x^3} dx = 2 \int_1^{\infty} x^{-2} dx \\ &= 2 \left[-\frac{1}{x} \right]_1^{\infty} = 2. \end{aligned}$$

Therefore, the mean is finite.

Step 3: Test the second moment.

$$E[X^2] = \int_1^{\infty} x^2 \frac{2}{x^3} dx = 2 \int_1^{\infty} \frac{dx}{x}.$$

For an upper cutoff b ,

$$2 \int_1^b \frac{dx}{x} = 2 \ln b \rightarrow \infty.$$

Thus the second moment diverges.

$$\boxed{E[X] = 2, \quad E[X^2] = \infty}$$

Consequently, $\text{Var}(X)$ is also infinite.