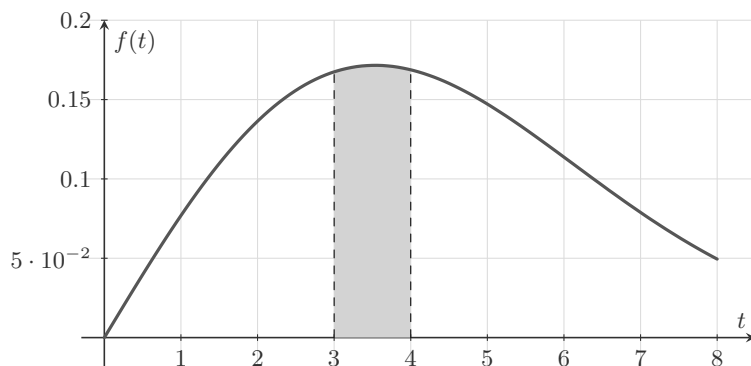
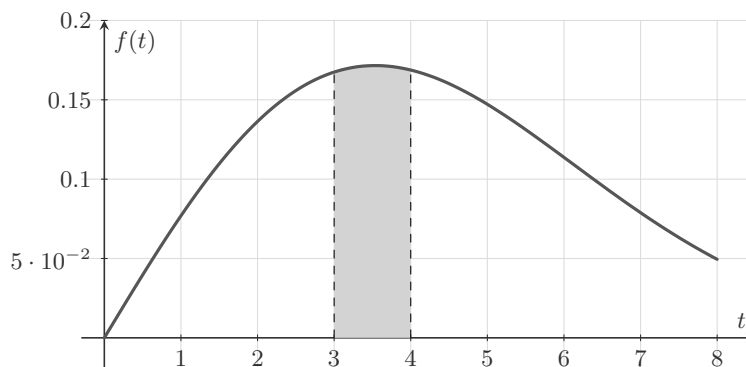


Question 8

A lifetime satisfies $P(X > t) = e^{-(t/5)^2}$, $t \geq 0$. Find its density and $P(3 < X < 4)$.



Question 8 – Solution



Step 1: Convert the survival function to the CDF. For $t \geq 0$,

$$P(X > t) = e^{-(t/5)^2} = e^{-t^2/25}.$$

Therefore,

$$F(t) = P(X \leq t) = 1 - P(X > t) = 1 - e^{-t^2/25}.$$

Step 2: Differentiate to obtain the density.

$$\begin{aligned} f(t) &= F'(t) \\ &= -\frac{d}{dt} \left(e^{-t^2/25} \right) \\ &= \frac{2t}{25} e^{-t^2/25}, \quad t \geq 0. \end{aligned}$$

Also, $f(t) = 0$ for $t < 0$.

Step 3: Compute the interval probability. Using the CDF,

$$\begin{aligned} P(3 < X < 4) &= F(4) - F(3) \\ &= \left(1 - e^{-16/25} \right) - \left(1 - e^{-9/25} \right) \\ &= e^{-9/25} - e^{-16/25}. \end{aligned}$$

$$P(3 < X < 4) = e^{-9/25} - e^{-16/25}$$

The same result follows by integrating $f(t)$ from 3 to 4.