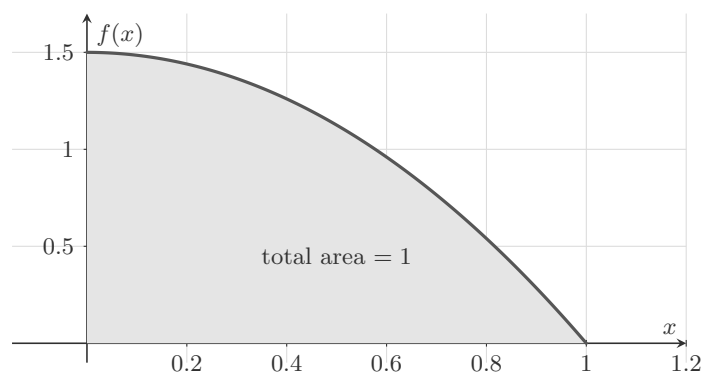
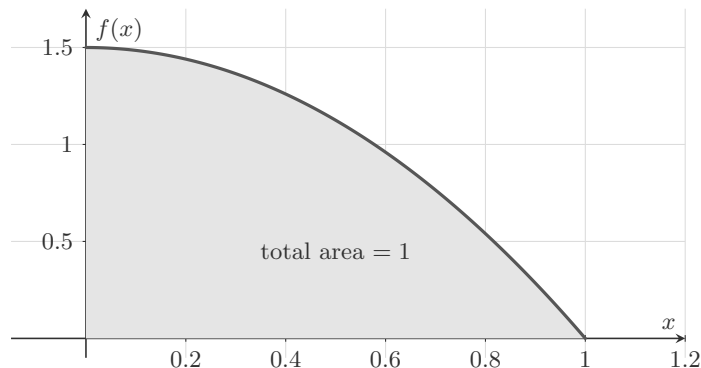


Question 7

A proposed density is $f(x) = \frac{3}{2}(1 - x^2)$ on $[0, 1]$. Is it valid? If not, repair only the constant.



Question 7 – Solution



Step 1: Check nonnegativity. For $0 \leq x \leq 1$,

$$1 - x^2 \geq 0,$$

so

$$f(x) = \frac{3}{2}(1 - x^2) \geq 0.$$

Step 2: Check the total area.

$$\begin{aligned} \int_0^1 f(x) dx &= \frac{3}{2} \int_0^1 (1 - x^2) dx \\ &= \frac{3}{2} \left[x - \frac{x^3}{3} \right]_0^1 \\ &= \frac{3}{2} \left(1 - \frac{1}{3} \right) = \frac{3}{2} \left(\frac{2}{3} \right) = 1. \end{aligned}$$

Step 3: State the conclusion. The function is nonnegative on its support and integrates to 1. Therefore it already satisfies both requirements for a probability density.

The proposed density is valid; no repair is needed.

If the constant had been unknown, normalization would give $c(2/3) = 1$, hence $c = 3/2$.