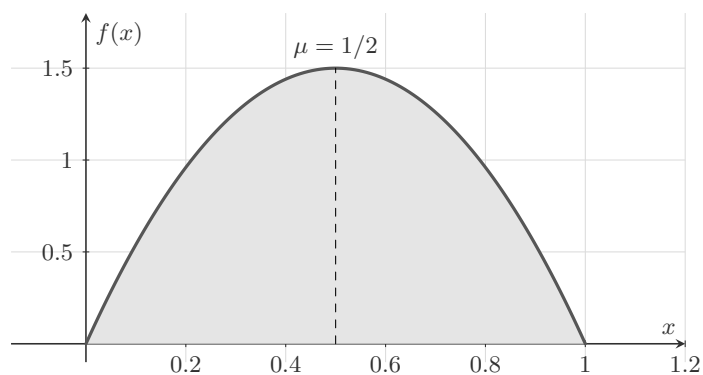
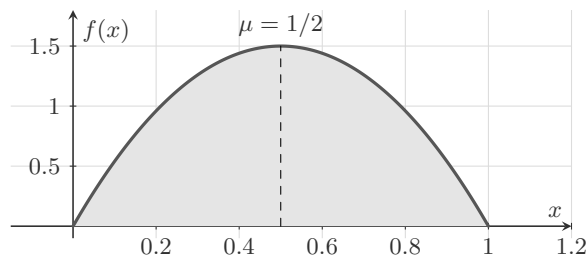


Question 6

For $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$, find $\text{Var}(X)$. Use symmetry for the mean.



Question 6 – Solution



Step 1: Use symmetry to find the mean. The density satisfies

$$f(1-x) = 6(1-x)x = f(x),$$

so it is symmetric about $x = 1/2$. Therefore,

$$\mu = E[X] = \frac{1}{2}.$$

Step 2: Compute the second moment.

$$\begin{aligned} E[X^2] &= \int_0^1 x^2 f(x) dx \\ &= \int_0^1 x^2 (6x - 6x^2) dx \\ &= \int_0^1 (6x^3 - 6x^4) dx \\ &= \left[\frac{3}{2}x^4 - \frac{6}{5}x^5 \right]_0^1 = \frac{3}{2} - \frac{6}{5} = \frac{3}{10}. \end{aligned}$$

Step 3: Apply the variance formula.

$$\begin{aligned} \text{Var}(X) &= E[X^2] - (E[X])^2 \\ &= \frac{3}{10} - \left(\frac{1}{2}\right)^2 \\ &= \frac{3}{10} - \frac{1}{4} = \frac{1}{20}. \end{aligned}$$

$$\text{Var}(X) = \frac{1}{20}$$