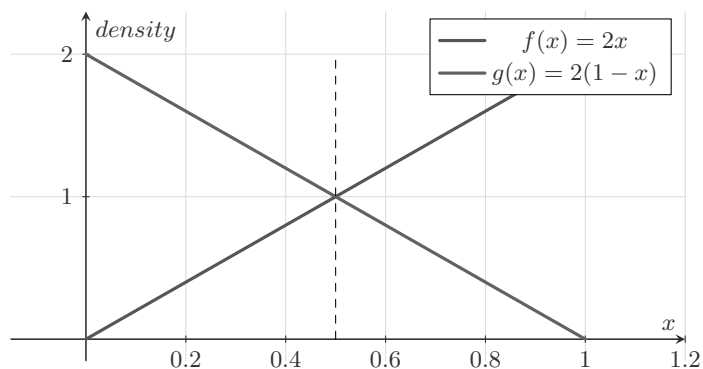
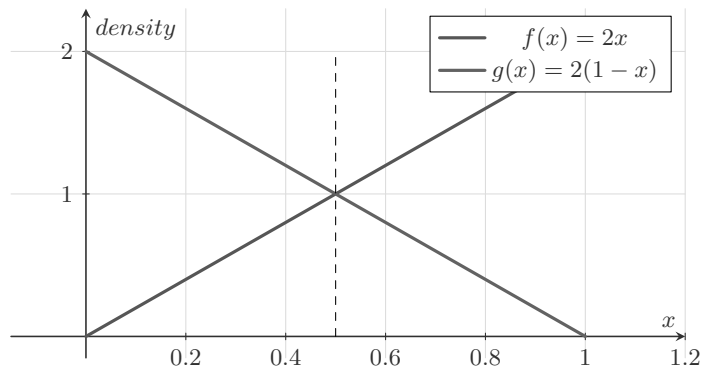


Question 5

Two densities on $[0, 1]$ are $f(x) = 2x$ and $g(x) = 2(1 - x)$. Compare their means without integration by identifying a reflection.



Question 5 – Solution



Step 1: Identify the reflection.

$$f(1-x) = 2(1-x) = g(x).$$

Therefore, the graph of g is the reflection of the graph of f across $x = 1/2$.

Step 2: Find the mean of f geometrically. The area under $f(x) = 2x$ is a right triangle with vertices

$$(0,0), \quad (1,0), \quad (1,2).$$

The x -coordinate of a triangle's centroid is the average of its vertex x -coordinates:

$$\mu_f = \frac{0 + 1 + 1}{3} = \frac{2}{3}.$$

Step 3: Reflect the mean. Reflection across $x = 1/2$ sends a location x to $1-x$. Hence

$$\mu_g = 1 - \mu_f = 1 - \frac{2}{3} = \frac{1}{3}.$$

$$\boxed{\mu_f = \frac{2}{3}, \quad \mu_g = \frac{1}{3}}$$

As a check, the two reflected means add to 1.