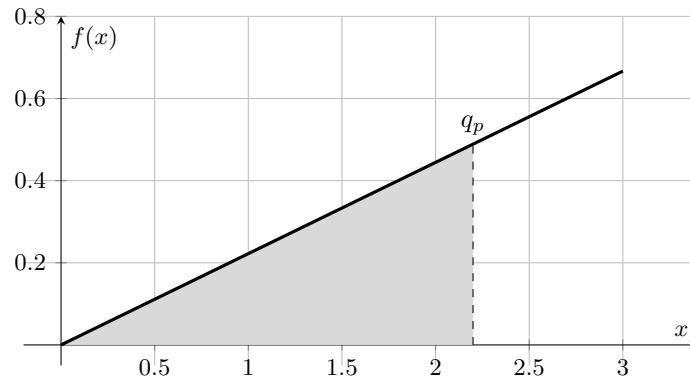
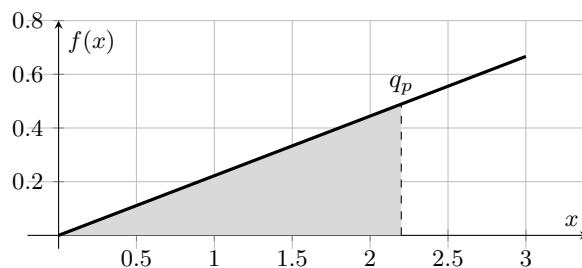


Question 4

A density is $f(x) = kx$ on $[0, a]$, where $a > 0$, and is zero elsewhere. Determine k , the CDF, and the p -quantile.



Question 4 – Solution



Step 1: Normalize the density.

$$1 = \int_0^a kx \, dx = k \left[\frac{x^2}{2} \right]_0^a = \frac{ka^2}{2}.$$

Hence

$$\boxed{k = \frac{2}{a^2}}.$$

Step 2: Build the CDF on the support. For $0 \leq x \leq a$,

$$\begin{aligned} F(x) &= \int_0^x \frac{2t}{a^2} \, dt \\ &= \frac{2}{a^2} \left[\frac{t^2}{2} \right]_0^x = \frac{x^2}{a^2}. \end{aligned}$$

Thus the complete CDF is

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x^2}{a^2}, & 0 \leq x \leq a, \\ 1, & x > a. \end{cases}$$

Step 3: Solve the quantile equation. The p -quantile q_p satisfies $F(q_p) = p$:

$$\frac{q_p^2}{a^2} = p.$$

Since $q_p \geq 0$,

$$\boxed{q_p = a\sqrt{p}}, \quad 0 \leq p \leq 1.$$