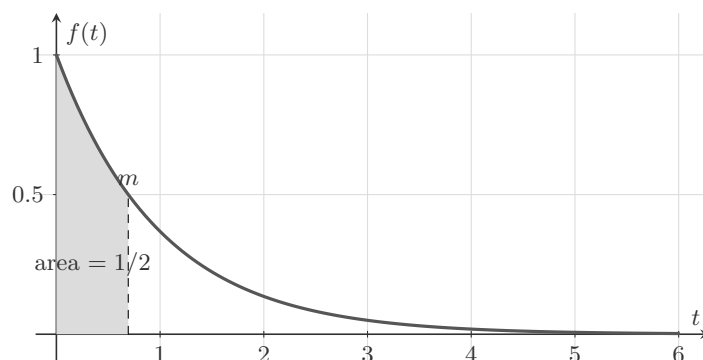
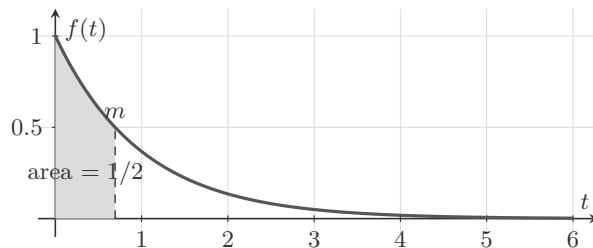


## Question 2

A waiting time has  $f(t) = \lambda e^{-\lambda t}$ ,  $t \geq 0$ . Find the median in terms of  $\lambda$ .



## Question 2 – Solution



**Step 1: Use the definition of a median.** The median  $m$  satisfies

$$P(T \leq m) = \frac{1}{2}.$$

Because  $T \geq 0$ ,

$$\int_0^m \lambda e^{-\lambda t} dt = \frac{1}{2}.$$

**Step 2: Evaluate the integral.**

$$\begin{aligned} \int_0^m \lambda e^{-\lambda t} dt &= [-e^{-\lambda t}]_0^m \\ &= 1 - e^{-\lambda m}. \end{aligned}$$

Therefore,

$$1 - e^{-\lambda m} = \frac{1}{2}.$$

**Step 3: Solve for  $m$ .**

$$\begin{aligned} e^{-\lambda m} &= \frac{1}{2}, \\ -\lambda m &= \ln\left(\frac{1}{2}\right) = -\ln 2, \\ m &= \frac{\ln 2}{\lambda}. \end{aligned}$$

$$\boxed{m = \frac{\ln 2}{\lambda}}$$

Since  $\lambda > 0$ , the median is positive as expected.