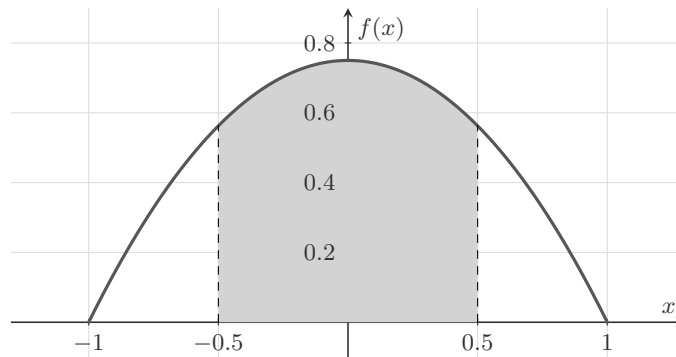
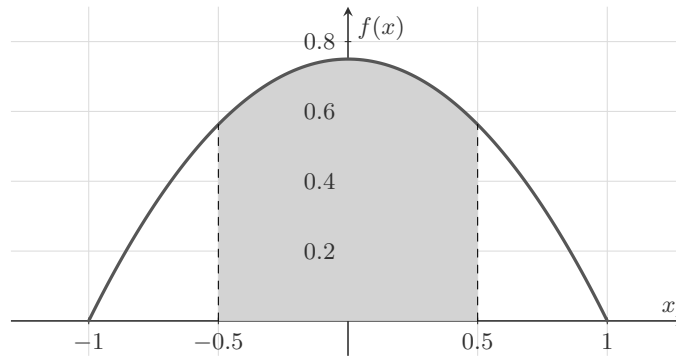


Question 1

A sensor error has density $f(x) = c(1 - x^2)$, $-1 \leq x \leq 1$. Find c and $P(|X| < 1/2)$.



Question 1 – Solution



Step 1: Use the normalization condition. A probability density must have total area 1:

$$1 = c \int_{-1}^1 (1 - x^2) dx.$$

Step 2: Evaluate the normalization integral.

$$\begin{aligned} \int_{-1}^1 (1 - x^2) dx &= \left[x - \frac{x^3}{3} \right]_{-1}^1 \\ &= \frac{2}{3} - \left(-\frac{2}{3} \right) = \frac{4}{3}. \end{aligned}$$

Thus

$$1 = c \left(\frac{4}{3} \right), \quad \boxed{c = \frac{3}{4}}.$$

Step 3: Translate the event.

$$|X| < \frac{1}{2} \iff -\frac{1}{2} < X < \frac{1}{2}.$$

Step 4: Integrate the density over this interval.

$$\begin{aligned} P\left(|X| < \frac{1}{2}\right) &= \frac{3}{4} \int_{-1/2}^{1/2} (1 - x^2) dx \\ &= \frac{3}{4} \left[x - \frac{x^3}{3} \right]_{-1/2}^{1/2} \\ &= \frac{3}{4} \left(\frac{11}{12} \right) = \frac{11}{16}. \end{aligned}$$

$$\boxed{P\left(|X| < \frac{1}{2}\right) = \frac{11}{16}}$$