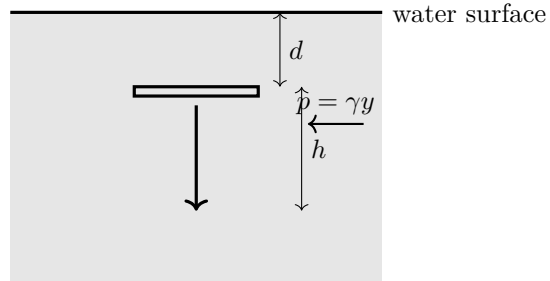
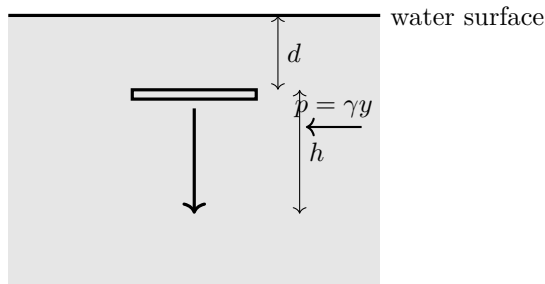


Question 10

A horizontal hatch of constant area $A > 0$ is slowly lowered from depth d to $d + h$, where $d, h > 0$. Assume an upward resisting hydrostatic force $F(y) = \gamma Ay$ at depth y , with constant weight density $\gamma > 0$. Find the work done against this force, ignoring other forces.



Question 10 – Solution



Step 1: State the force model. Assume the hydrostatic force resisting the downward motion has magnitude

$$F(y) = p(y)A = \gamma yA,$$

where y is the hatch depth. The area A is constant.

Step 2: Write a differential amount of work. Moving the hatch through a small downward distance dy requires

$$dW = F(y) dy = \gamma Ay dy.$$

Step 3: Integrate over the change in depth. The hatch moves from $y = d$ to $y = d + h$, so

$$\begin{aligned} W &= \int_d^{d+h} \gamma Ay dy \\ &= \gamma A \left[\frac{y^2}{2} \right]_d^{d+h} \\ &= \frac{\gamma A}{2} ((d+h)^2 - d^2). \end{aligned}$$

Step 4: Simplify.

$$\begin{aligned} W &= \frac{\gamma A}{2} (d^2 + 2dh + h^2 - d^2) \\ &= \gamma A \left(dh + \frac{h^2}{2} \right). \end{aligned}$$

$$\boxed{W = \gamma A \left(dh + \frac{h^2}{2} \right)}$$

This is the external work under the stated resisting-force assumption.