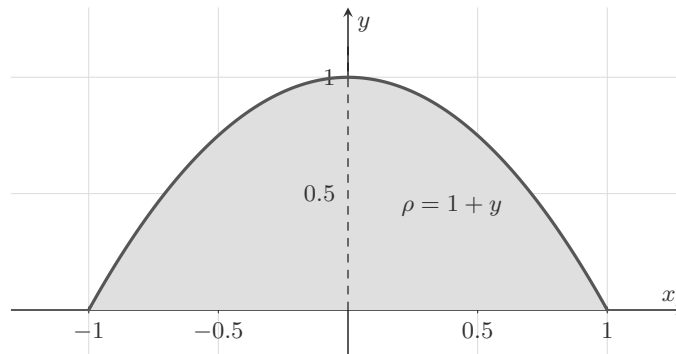
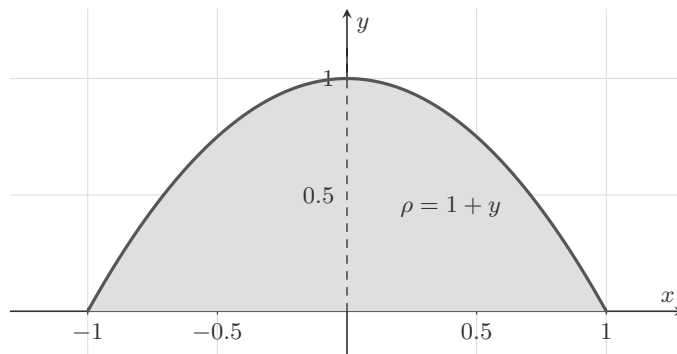


Question 10

A symmetric lamina has density $\rho(x, y) = 1 + y$ on $-1 \leq x \leq 1, 0 \leq y \leq 1 - x^2$. State $\bar{x}$, then set up exact integrals for m and $\bar{y}$.



Question 10 – Solution



Step 1: Use symmetry. The region is symmetric about the y -axis, and $\rho(x, y) = 1 + y$ is unchanged when x is replaced by $-x$. Therefore,

$$\boxed{\bar{x} = 0}.$$

Step 2: Set up and evaluate the mass. Let $h = 1 - x^2$. Then

$$\begin{aligned} m &= \int_{-1}^1 \int_0^{1-x^2} (1+y) \, dy \, dx \\ &= \int_{-1}^1 \left[h + \frac{h^2}{2} \right] dx \\ &= \int_{-1}^1 \left[(1-x^2) + \frac{1}{2}(1-x^2)^2 \right] dx = \frac{28}{15}. \end{aligned}$$

Step 3: Set up and evaluate the moment about the x -axis.

$$\begin{aligned} M_x &= \int_{-1}^1 \int_0^{1-x^2} y(1+y) \, dy \, dx \\ &= \int_{-1}^1 \left[\frac{h^2}{2} + \frac{h^3}{3} \right] dx \\ &= \frac{1}{2} \int_{-1}^1 (1-x^2)^2 dx + \frac{1}{3} \int_{-1}^1 (1-x^2)^3 dx \\ &= \frac{8}{15} + \frac{32}{105} = \frac{88}{105}. \end{aligned}$$

Step 4: Compute $\bar{y}$.

$$\bar{y} = \frac{M_x}{m} = \frac{\frac{88}{105}}{\frac{28}{15}} = \frac{22}{49}.$$

$$\boxed{(\bar{x}, \bar{y}) = \left(0, \frac{22}{49} \right)}$$