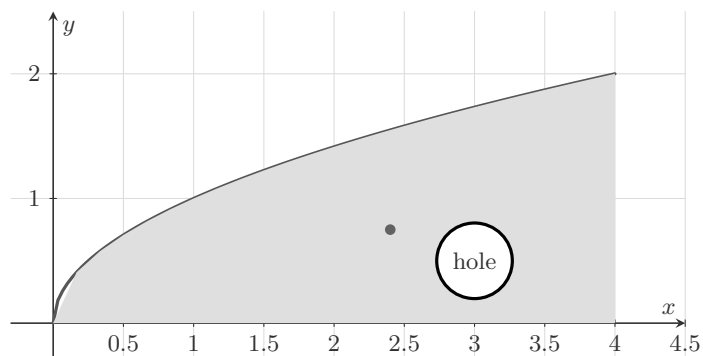
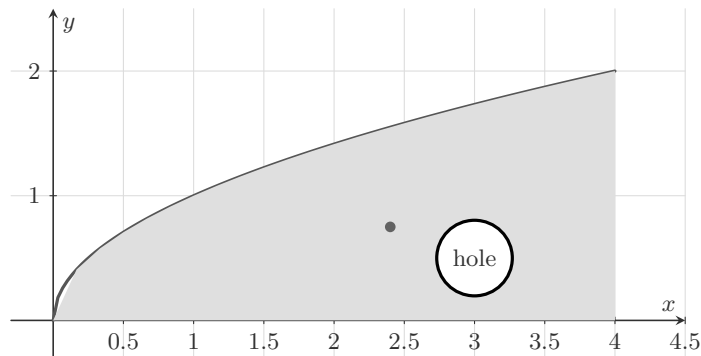


Question 9

A lamina is bounded by $y = \sqrt{x}$, $y = 0$, $x = 4$. A circular hole of radius $(1/2)$, centered at $((3, 1/2))$, is removed. Find the new centroid using subtraction.



Question 9 – Solution



Step 1: Find the area of the original region.

$$A_0 = \int_0^4 \sqrt{x} \, dx = \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{16}{3}.$$

Step 2: Find the original moments.

$$M_{y,0} = \int_0^4 x \sqrt{x} \, dx = \left[\frac{2}{5} x^{5/2} \right]_0^4 = \frac{64}{5},$$

$$M_{x,0} = \frac{1}{2} \int_0^4 (\sqrt{x})^2 \, dx = \frac{1}{2} \int_0^4 x \, dx = 4.$$

Hence the original centroid is

$$(\bar{x}_0, \bar{y}_0) = \left(\frac{M_{y,0}}{A_0}, \frac{M_{x,0}}{A_0} \right) = \left(\frac{12}{5}, \frac{3}{4} \right).$$

Step 3: Record the hole's area and moments. The hole has radius $1/2$ and centroid $(3, 1/2)$, so

$$A_h = \pi \left(\frac{1}{2} \right)^2 = \frac{\pi}{4}, \quad M_{y,h} = 3 \left(\frac{\pi}{4} \right) = \frac{3\pi}{4}, \quad M_{x,h} = \frac{1}{2} \left(\frac{\pi}{4} \right) = \frac{\pi}{8}.$$

Step 4: Subtract the hole.

$$A = \frac{16}{3} - \frac{\pi}{4}, \quad M_y = \frac{64}{5} - \frac{3\pi}{4}, \quad M_x = 4 - \frac{\pi}{8}.$$

Step 5: Divide the remaining moments by the remaining area.

$$\bar{x} = \frac{\frac{64}{5} - \frac{3\pi}{4}}{\frac{16}{3} - \frac{\pi}{4}}, \quad \bar{y} = \frac{4 - \frac{\pi}{8}}{\frac{16}{3} - \frac{\pi}{4}}$$