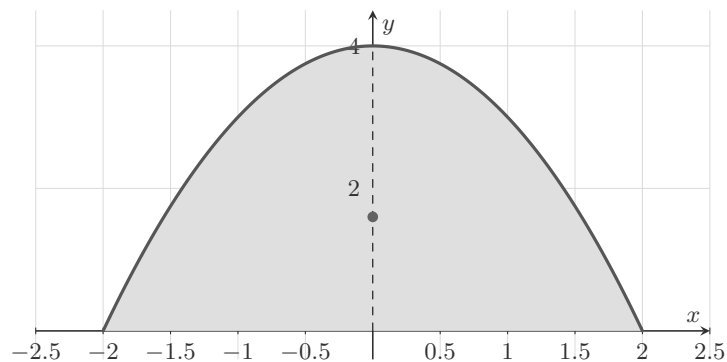
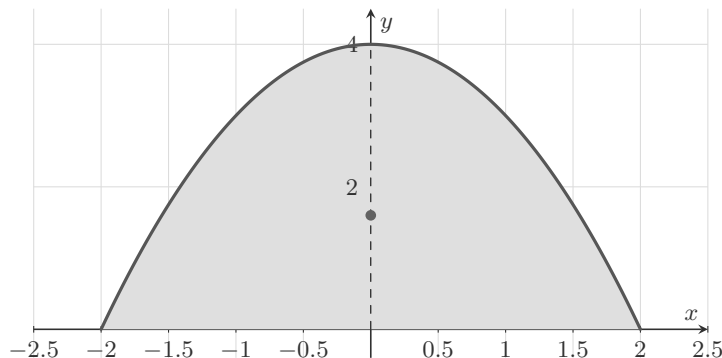


Question 7

A plate under $y = 4 - x^2$, $-2 \leq x \leq 2$, has uniform density. Use symmetry before integration; find its centroid.



Question 7 – Solution



Step 1: Use symmetry before integrating. Both the region and its uniform density are symmetric about the y -axis. Therefore,

$$\bar{x} = 0.$$

Step 2: Find the area.

$$\begin{aligned} A &= \int_{-2}^2 (4 - x^2) dx = 2 \int_0^2 (4 - x^2) dx \\ &= 2 \left[4x - \frac{x^3}{3} \right]_0^2 = 2 \left(8 - \frac{8}{3} \right) = \frac{32}{3}. \end{aligned}$$

Step 3: Find the moment about the x -axis. For a vertical strip under $f(x) = 4 - x^2$,

$$\begin{aligned} M_x &= \frac{1}{2} \int_{-2}^2 f(x)^2 dx = \int_0^2 (4 - x^2)^2 dx \\ &= \int_0^2 (16 - 8x^2 + x^4) dx \\ &= \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \frac{256}{15}. \end{aligned}$$

Step 4: Compute $\bar{y}$.

$$\bar{y} = \frac{M_x}{A} = \frac{\frac{256}{15}}{\frac{32}{3}} = \frac{8}{5}.$$

$$\boxed{(\bar{x}, \bar{y}) = \left(0, \frac{8}{5} \right)}$$