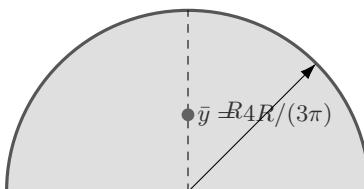
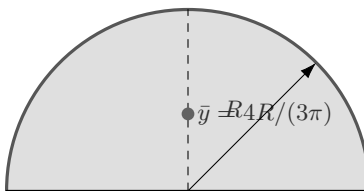


Question 4

A uniform semicircular lamina $x^2 + y^2 \leq R^2, y \geq 0$ has $\bar{x} = 0$. Derive $\bar{y}$.



Question 4 – Solution



Step 1: Use symmetry. The lamina is symmetric about the y -axis, so

$$\bar{x} = 0.$$

Step 2: Find the area. The region is one-half of a disk of radius R :

$$A = \frac{1}{2}\pi R^2.$$

Step 3: Write the upper boundary. For $-R \leq x \leq R$,

$$f(x) = \sqrt{R^2 - x^2}.$$

A vertical strip of height $f(x)$ has centroid height $f(x)/2$.

Step 4: Find the moment about the x -axis.

$$\begin{aligned} M_x &= \frac{1}{2} \int_{-R}^R f(x)^2 dx = \frac{1}{2} \int_{-R}^R (R^2 - x^2) dx \\ &= \frac{1}{2} \left[R^2 x - \frac{x^3}{3} \right]_{-R}^R = \frac{2R^3}{3}. \end{aligned}$$

Step 5: Divide the moment by the area.

$$\bar{y} = \frac{M_x}{A} = \frac{\frac{2R^3}{3}}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}.$$

$$\boxed{(\bar{x}, \bar{y}) = \left(0, \frac{4R}{3\pi}\right)}$$