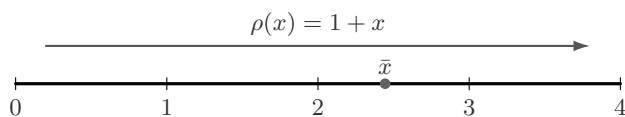
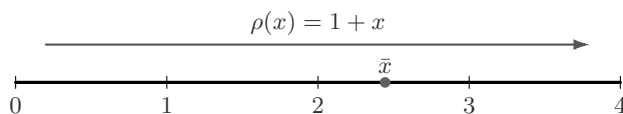


Question 3

A 4-m rod on $0 \leq x \leq 4$ has density $\rho(x) = k(1 + x)$. Its mass is (12) kg. Find k and its balance point.



Question 3 – Solution



Step 1: Use the total mass to determine k .

$$\begin{aligned} 12 &= \int_0^4 \rho(x) dx = k \int_0^4 (1 + x) dx \\ &= k \left[x + \frac{x^2}{2} \right]_0^4 = k(4 + 8) = 12k. \end{aligned}$$

Therefore, $k = 1$, so $\rho(x) = 1 + x$.

Step 2: Find the first moment about the origin.

$$\begin{aligned} M_0 &= \int_0^4 x\rho(x) dx = \int_0^4 (x + x^2) dx \\ &= \left[\frac{x^2}{2} + \frac{x^3}{3} \right]_0^4 = 8 + \frac{64}{3} = \frac{88}{3}. \end{aligned}$$

Step 3: Divide the moment by the mass.

$$\bar{x} = \frac{M_0}{m} = \frac{\frac{88}{3}}{12} = \frac{22}{9} \text{ m}.$$

The balance point lies to the right of the midpoint because the density increases with x .

$k = 1, \quad \bar{x} = \frac{22}{9} \text{ m}$
