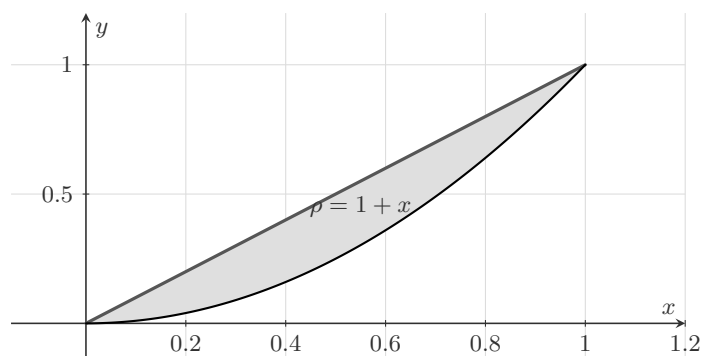
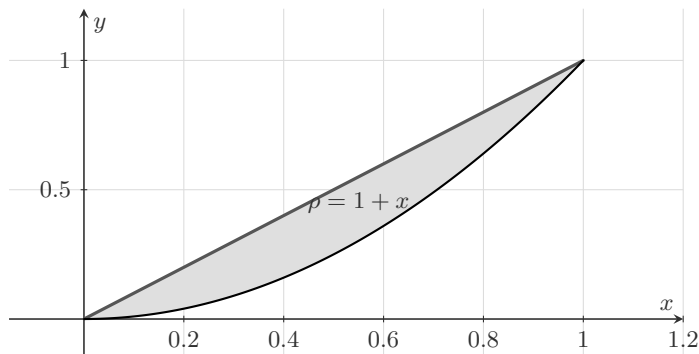


Question 2

A lamina lies between $y = x$ and $y = x^2$, $0 \leq x \leq 1$, with density $\rho(x) = 1 + x$. Find its mass and $\bar{x}$.



Question 2 – Solution



Step 1: Identify the strip dimensions. On $0 \leq x \leq 1$, the upper curve is $y = x$ and the lower curve is $y = x^2$. Thus

$$dA = (x - x^2) dx, \quad dm = \rho(x) dA = (1 + x)(x - x^2) dx.$$

Step 2: Find the mass. Since $(1 + x)(x - x^2) = x - x^3$,

$$m = \int_0^1 (x - x^3) dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{4}.$$

Step 3: Find the moment about the y-axis. The moment arm is x , so

$$\begin{aligned} M_y &= \int_0^1 x dm = \int_0^1 x(x - x^3) dx \\ &= \int_0^1 (x^2 - x^4) dx = \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1 = \frac{2}{15}. \end{aligned}$$

Step 4: Compute the x-coordinate of the center of mass.

$$\bar{x} = \frac{M_y}{m} = \frac{\frac{2}{15}}{\frac{1}{4}} = \frac{8}{15}.$$

$m = \frac{1}{4}, \quad \bar{x} = \frac{8}{15}$
