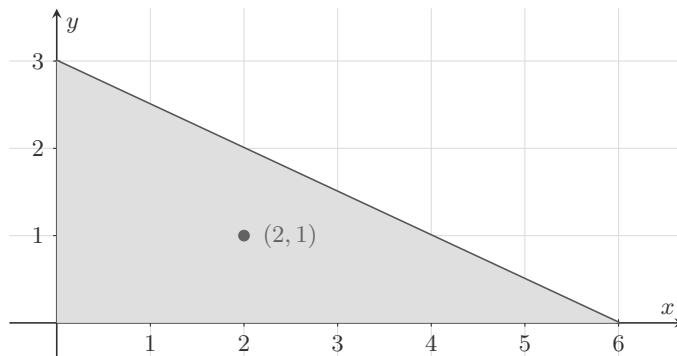


Question 1

A triangular sign occupies $x \geq 0, y \geq 0, x + 2y \leq 6$. Find its centroid using integrals.



Question 1 – Solution



Step 1: Write the upper boundary as a function of x . From $x + 2y = 6$,

$$y = f(x) = 3 - \frac{x}{2}, \quad 0 \leq x \leq 6.$$

Step 2: Find the area (and hence the mass for unit density).

$$\begin{aligned} A &= \int_0^6 f(x) dx = \int_0^6 \left(3 - \frac{x}{2}\right) dx \\ &= \left[3x - \frac{x^2}{4}\right]_0^6 = 18 - 9 = 9. \end{aligned}$$

Step 3: Find the moment about the y -axis.

$$\begin{aligned} M_y &= \int_0^6 x f(x) dx = \int_0^6 \left(3x - \frac{x^2}{2}\right) dx \\ &= \left[\frac{3x^2}{2} - \frac{x^3}{6}\right]_0^6 = 54 - 36 = 18. \end{aligned}$$

Step 4: Find the moment about the x -axis. Each vertical strip has centroid height $f(x)/2$, so

$$M_x = \frac{1}{2} \int_0^6 f(x)^2 dx = \frac{1}{2} \int_0^6 \left(3 - \frac{x}{2}\right)^2 dx = 9.$$

Step 5: Divide each moment by the area.

$$\bar{x} = \frac{M_y}{A} = \frac{18}{9} = 2, \quad \bar{y} = \frac{M_x}{A} = \frac{9}{9} = 1.$$

$$\boxed{(\bar{x}, \bar{y}) = (2, 1)}$$