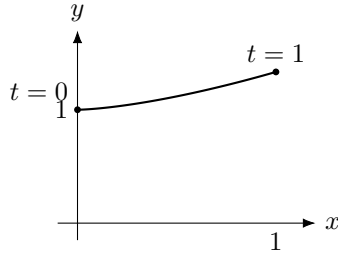


Question 9

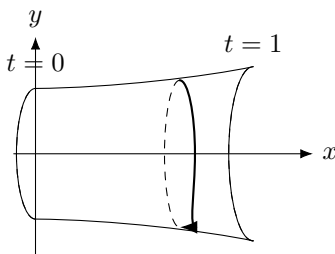
Parametric curve: $x = t^2$, $y = 1 + t^3/3$, for $0 \leq t \leq 1$.

Axis of rotation: the x -axis.

Task: Set up, but do not evaluate, the surface-area integral.



Question 9 – Solution



Step 1: Differentiate both coordinates.

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = t^2.$$

Step 2: Find the parametric arc-length factor.

$$\begin{aligned} ds &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ &= \sqrt{4t^2 + t^4} dt \\ &= t\sqrt{4 + t^2} dt \end{aligned}$$

because $t \geq 0$.

Step 3: Use radius $y = 1 + t^3/3$.

$$\begin{aligned} S &= 2\pi \int_0^1 y \, ds \\ &= 2\pi \int_0^1 \left(1 + \frac{t^3}{3}\right) t\sqrt{4 + t^2} \, dt \\ &= 2\pi \int_0^1 \left(t + \frac{t^4}{3}\right) \sqrt{4 + t^2} \, dt. \end{aligned}$$

$$\boxed{S = 2\pi \int_0^1 \left(t + \frac{t^4}{3}\right) \sqrt{4 + t^2} \, dt}$$