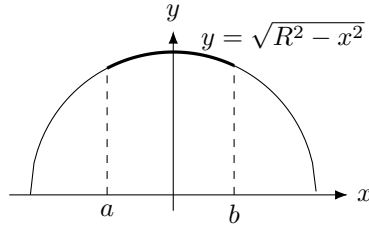


Question 8

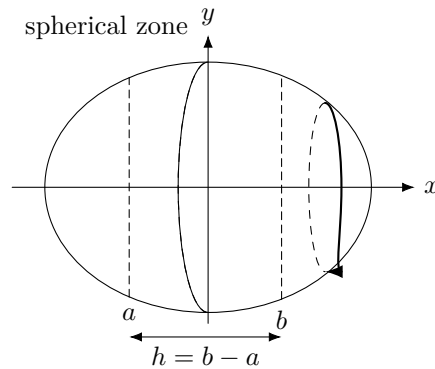
Curve: $y = \sqrt{R^2 - x^2}$ on $a \leq x \leq b$, where $R > 0$ and $-R \leq a < b \leq R$.

Axis of rotation: the x -axis.

Task: For fixed R , show that the surface area depends only on the zone height $h = b - a$.



Question 8 – Solution



Step 1: Differentiate.

$$y' = -\frac{x}{\sqrt{R^2 - x^2}}.$$

Step 2: Simplify the arc-length factor.

$$\begin{aligned}\sqrt{1 + (y')^2} &= \sqrt{1 + \frac{x^2}{R^2 - x^2}} \\ &= \sqrt{\frac{R^2}{R^2 - x^2}} = \frac{R}{\sqrt{R^2 - x^2}}.\end{aligned}$$

Step 3: Calculate the surface area.

$$\begin{aligned}S &= 2\pi \int_a^b y \sqrt{1 + (y')^2} dx \\ &= 2\pi \int_a^b \sqrt{R^2 - x^2} \frac{R}{\sqrt{R^2 - x^2}} dx \\ &= 2\pi R \int_a^b dx \\ &= 2\pi R(b - a) = 2\pi Rh.\end{aligned}$$

$$\boxed{S = 2\pi R(b - a) = 2\pi Rh}$$