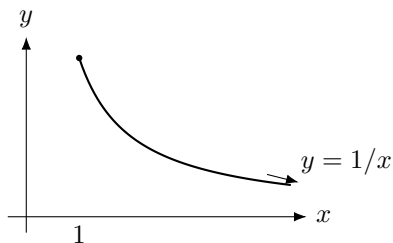


Question 7

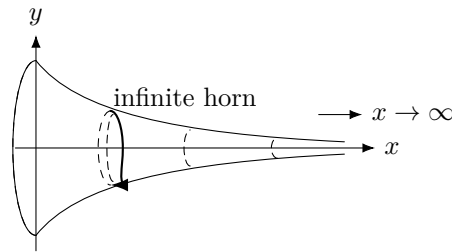
Curve: $y = 1/x$ on $x \geq 1$.

Axis of rotation: the x -axis.

Task: Determine whether the generated surface has finite area.



Question 7 – Solution



Step 1: Differentiate and set up the improper integral.

$$y' = -\frac{1}{x^2}.$$

Thus

$$S = 2\pi \int_1^{\infty} \frac{1}{x} \sqrt{1 + \frac{1}{x^4}} dx.$$

Step 2: Find a lower comparison. Since

$$\sqrt{1 + x^{-4}} > 1,$$

we have

$$\frac{1}{x} \sqrt{1 + x^{-4}} > \frac{1}{x}.$$

Step 3: Test the comparison integral.

$$\int_1^{\infty} \frac{dx}{x} = \lim_{b \rightarrow \infty} \ln b = \infty.$$

By direct comparison, the surface-area integral diverges.

$S = \infty$ (infinite surface area)
