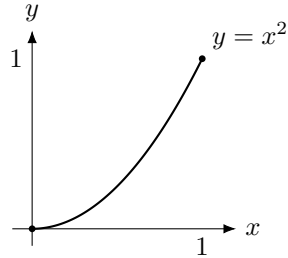


Question 6

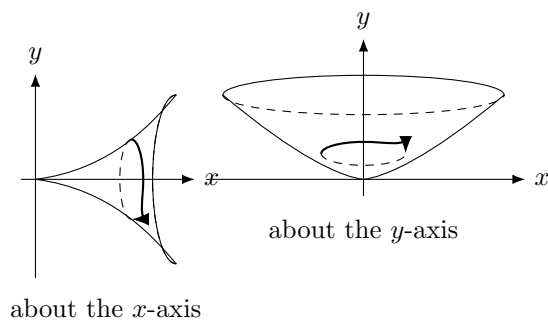
Curve: $y = x^2$ on $0 \leq x \leq 1$.

Axes of rotation: first the x -axis, then the y -axis.

Task: Write both surface-area integrals and determine which is larger without evaluating either integral.



Question 6 – Solution



Step 1: Find the common arc-length factor.

$$y' = 2x, \quad ds = \sqrt{1 + 4x^2} \, dx.$$

Step 2: Rotate about the x -axis. The radius is $y = x^2$:

$$S_x = 2\pi \int_0^1 x^2 \sqrt{1 + 4x^2} \, dx.$$

Step 3: Rotate about the y -axis. The radius is x :

$$S_y = 2\pi \int_0^1 x \sqrt{1 + 4x^2} \, dx.$$

Step 4: Compare the integrands. For $0 < x < 1$,

$$x > x^2.$$

Multiplying by the positive common factor $2\pi\sqrt{1 + 4x^2}$ preserves the inequality. Therefore,

$$\boxed{S_y > S_x}.$$