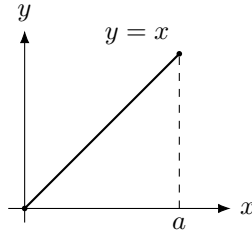


Question 5

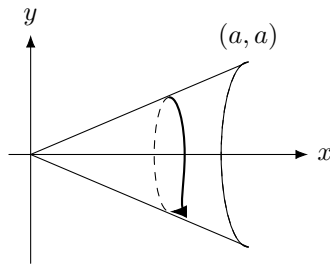
Curve: $y = x$ on $0 \leq x \leq a$, where $a > 0$.

Axis of rotation: the x -axis.

Task: Find a so that the generated surface area is $9\pi\sqrt{2}$.



Question 5 – Solution



Step 1: Find the arc-length factor.

$$y' = 1, \quad \sqrt{1 + (y')^2} = \sqrt{2}.$$

Step 2: Write and evaluate the area.

$$\begin{aligned} S &= 2\pi \int_0^a y \sqrt{1 + (y')^2} dx \\ &= 2\pi \int_0^a x \sqrt{2} dx \\ &= 2\pi \sqrt{2} \left[\frac{x^2}{2} \right]_0^a = \pi \sqrt{2} a^2. \end{aligned}$$

Step 3: Set the area equal to $9\pi\sqrt{2}$ and solve.

$$\begin{aligned} \pi \sqrt{2} a^2 &= 9\pi \sqrt{2}, \\ a^2 &= 9, \\ a &= 3 \end{aligned}$$

because $a > 0$.

$$\boxed{a = 3}$$