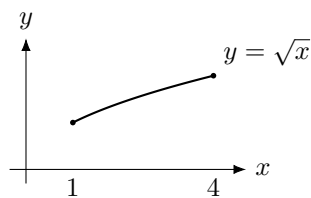


Question 4

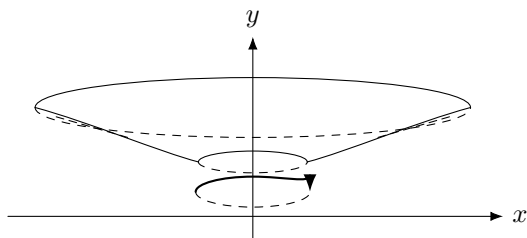
Curve: $y = \sqrt{x}$ on $1 \leq x \leq 4$.

Axis of rotation: the y -axis.

Task: Find the exact surface area.



Question 4 – Solution



Step 1: Find the radius and derivative. Rotation about the y -axis gives radius $r = x$, and

$$y' = \frac{1}{2\sqrt{x}}.$$

Step 2: Set up and simplify.

$$\begin{aligned} S &= 2\pi \int_1^4 x \sqrt{1 + \frac{1}{4x}} \, dx \\ &= \pi \int_1^4 \sqrt{x(4x+1)} \, dx. \end{aligned}$$

Step 3: Substitute $u = 8x + 1$. Then

$$dx = \frac{du}{8}, \quad \sqrt{x(4x+1)} = \frac{1}{4} \sqrt{u^2 - 1}.$$

Thus

$$S = \frac{\pi}{32} \int_9^{33} \sqrt{u^2 - 1} \, du.$$

Step 4: Integrate and return to x .

$$\int \sqrt{u^2 - 1} \, du = \frac{1}{2} \left(u \sqrt{u^2 - 1} - \ln(u + \sqrt{u^2 - 1}) \right).$$

Therefore,

$$S = \pi \left[\frac{(8x+1)\sqrt{x(4x+1)}}{16} - \frac{1}{64} \ln \left(8x+1 + 4\sqrt{x(4x+1)} \right) \right]_1^4$$