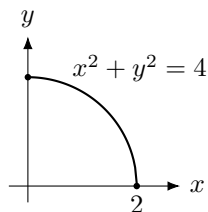


Question 3

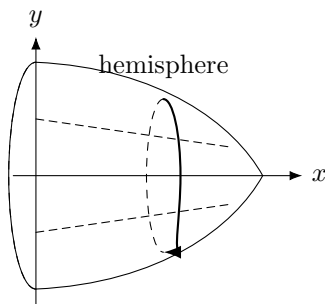
Curve: $y = \sqrt{4 - x^2}$ on $0 \leq x \leq 2$.

Axis of rotation: the x -axis.

Task: Identify the resulting surface and find its area.



Question 3 – Solution



Step 1: Identify the surface. The curve is the upper-right quarter of the circle $x^2 + y^2 = 4$. Rotating it about the x -axis produces a hemisphere of radius (2).

Step 2: Differentiate.

$$y' = -\frac{x}{\sqrt{4-x^2}}.$$

Step 3: Simplify the arc-length factor.

$$\begin{aligned}\sqrt{1+(y')^2} &= \sqrt{1 + \frac{x^2}{4-x^2}} \\ &= \sqrt{\frac{4}{4-x^2}} = \frac{2}{\sqrt{4-x^2}}.\end{aligned}$$

Step 4: Calculate the surface area.

$$\begin{aligned}S &= 2\pi \int_0^2 y \sqrt{1+(y')^2} dx \\ &= 2\pi \int_0^2 \sqrt{4-x^2} \frac{2}{\sqrt{4-x^2}} dx \\ &= 4\pi \int_0^2 dx = 8\pi.\end{aligned}$$

hemisphere, $S = 8\pi$
