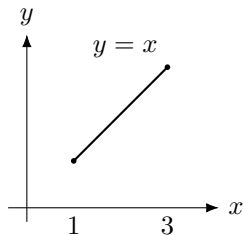


Question 1

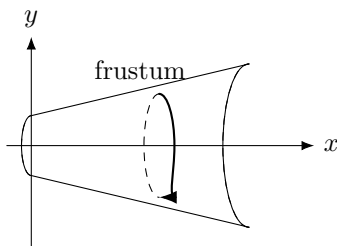
Curve: $y = x$ on $1 \leq x \leq 3$.

Axis of rotation: the x -axis.

Task: Find the lateral surface area, then verify the result with the frustum formula.



Question 1 – Solution



Step 1: Differentiate and find ds .

$$y' = 1, \quad ds = \sqrt{1 + (y')^2} dx = \sqrt{2} dx.$$

Step 2: Apply the surface-area formula.

$$\begin{aligned} S &= 2\pi \int_1^3 y \, ds \\ &= 2\pi \int_1^3 x\sqrt{2} \, dx \\ &= 2\pi\sqrt{2} \left[\frac{x^2}{2} \right]_1^3 \\ &= \pi\sqrt{2}(9 - 1) = 8\pi\sqrt{2}. \end{aligned}$$

Step 3: Verify with the frustum formula. The radii are $r_1 = 1$ and $r_2 = 3$. The slant height is

$$s = \sqrt{(3 - 1)^2 + (3 - 1)^2} = 2\sqrt{2}.$$

Therefore,

$$S = \pi(r_1 + r_2)s = \pi(1 + 3)(2\sqrt{2}) = 8\pi\sqrt{2}.$$

$$\boxed{S = 8\pi\sqrt{2}}$$