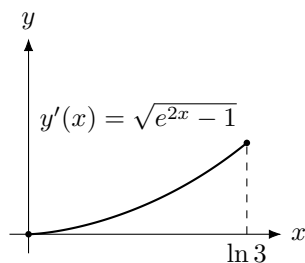


Question 10

A curve satisfies

$$y'(x) = \sqrt{e^{2x} - 1}, \quad 0 \leq x \leq \ln 3.$$

Find its length. A formula for y is not needed.



Question 10 – Solution

Step 1: Substitute the derivative into the arc-length formula.

$$\begin{aligned}1 + (y')^2 &= 1 + \left(\sqrt{e^{2x} - 1}\right)^2 \\&= 1 + e^{2x} - 1 = e^{2x}.\end{aligned}$$

Step 2: Simplify the square root. Since $e^x > 0$,

$$\sqrt{1 + (y')^2} = \sqrt{e^{2x}} = e^x.$$

Step 3: Integrate and evaluate.

$$\begin{aligned}L &= \int_0^{\ln 3} e^x \, dx \\&= [e^x]_0^{\ln 3} \\&= e^{\ln 3} - e^0 = 3 - 1 = 2.\end{aligned}$$

$$\boxed{L = 2}$$