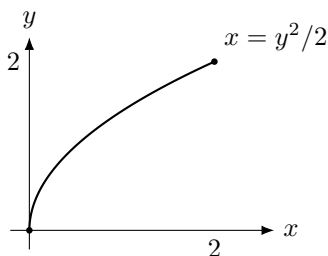


Question 9

For

$$x = \frac{y^2}{2}, \quad 0 \leq y \leq 2,$$

find the length using y . Then write the equivalent x -integral without evaluating it.



Question 9 – Solution

Step 1: Differentiate with respect to y .

$$\frac{dx}{dy} = y.$$

Step 2: Set up and evaluate the y -integral.

$$\begin{aligned} L &= \int_0^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \\ &= \int_0^2 \sqrt{1 + y^2} dy \\ &= \left[\frac{y}{2} \sqrt{1 + y^2} + \frac{1}{2} \operatorname{arsinh} y \right]_0^2 \\ &= \sqrt{5} + \frac{1}{2} \operatorname{arsinh} 2. \end{aligned}$$

$$L = \sqrt{5} + \frac{1}{2} \operatorname{arsinh} 2$$

Step 3: Rewrite using x . Since $y = \sqrt{2x}$,

$$\frac{dy}{dx} = \frac{1}{\sqrt{2x}}.$$

The x -values run from 0 to 2, so

$$L = \int_0^2 \sqrt{1 + \frac{1}{2x}} dx .$$