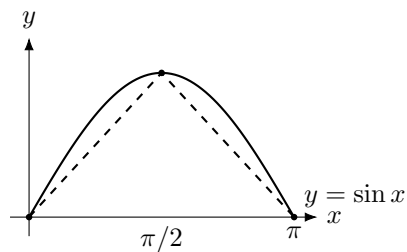


Question 8

Use two straight chords through $x = 0, \pi/2, \pi$ to estimate the length of

$$y = \sin x, \quad 0 \leq x \leq \pi.$$

Explain why the estimate is too small.



Question 8 – Solution

Step 1: List the three points.

$$(0, 0), \quad \left(\frac{\pi}{2}, 1\right), \quad (\pi, 0).$$

Step 2: Find the first chord length.

$$\begin{aligned} d_1 &= \sqrt{\left(\frac{\pi}{2} - 0\right)^2 + (1 - 0)^2} \\ &= \sqrt{\frac{\pi^2}{4} + 1}. \end{aligned}$$

By symmetry, $d_2 = d_1$. Thus

$$L_{\text{chord}} = 2d_1 = 2\sqrt{\frac{\pi^2}{4} + 1} = \sqrt{\pi^2 + 4}.$$

Step 3: Compare with the true length. The true arc length is

$$L = \int_0^\pi \sqrt{1 + \cos^2 x} \, dx.$$

A curved arc between two points is longer than its straight chord. Since the sine curve is not straight on either half,

$L_{\text{chord}} = \sqrt{\pi^2 + 4}, \quad L > L_{\text{chord}}$
