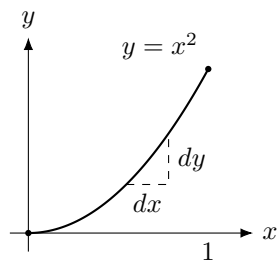


Question 7

A student writes

$$L = \int_0^1 (1 + y') \, dx$$

for $y = x^2$. Explain the error and find the correct length.



Question 7 – Solution

Step 1: Correct the geometric formula. A small arc segment is the hypotenuse of a right triangle:

$$ds^2 = dx^2 + dy^2.$$

Since $dy = y' dx$,

$$ds = \sqrt{dx^2 + (y' dx)^2} = \sqrt{1 + (y')^2} dx.$$

The student's expression adds horizontal and vertical changes instead of using the Pythagorean theorem.

Step 2: Differentiate and set up the length.

$$y' = 2x, \quad L = \int_0^1 \sqrt{1 + 4x^2} dx.$$

Step 3: Evaluate.

$$\begin{aligned} L &= \left[\frac{x}{2} \sqrt{1 + 4x^2} + \frac{1}{4} \ln(2x + \sqrt{1 + 4x^2}) \right]_0^1 \\ &= \frac{\sqrt{5}}{2} + \frac{1}{4} \ln(2 + \sqrt{5}). \end{aligned}$$

$$L = \frac{\sqrt{5}}{2} + \frac{1}{4} \ln(2 + \sqrt{5})$$