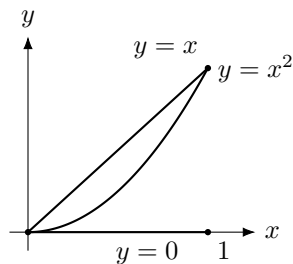


Question 6

Rank the arc lengths on $0 \leq x \leq 1$:

$$y = 0, \quad y = x, \quad y = x^2.$$



Question 6 – Solution

Step 1: Find the first two lengths.

$$L_0 = \int_0^1 \sqrt{1 + 0^2} \, dx = 1,$$

$$L_x = \int_0^1 \sqrt{1 + 1^2} \, dx = \sqrt{2} \approx 1.414.$$

Step 2: Set up the third length. For $y = x^2$, $y' = 2x$, so

$$L_{x^2} = \int_0^1 \sqrt{1 + 4x^2} \, dx.$$

Use the standard antiderivative

$$\int \sqrt{1 + 4x^2} \, dx = \frac{x}{2} \sqrt{1 + 4x^2} + \frac{1}{4} \operatorname{arsinh}(2x).$$

Therefore,

$$\begin{aligned} L_{x^2} &= \left[\frac{x}{2} \sqrt{1 + 4x^2} + \frac{1}{4} \operatorname{arsinh}(2x) \right]_0^1 \\ &= \frac{\sqrt{5}}{2} + \frac{1}{4} \operatorname{arsinh} 2 \\ &\approx 1.479. \end{aligned}$$

Step 3: Rank the values.

$$\boxed{L_0 < L_x < L_{x^2}}$$