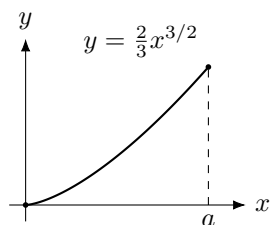


Question 5

Find $a > 0$ so that

$$y = \frac{2}{3}x^{3/2}, \quad 0 \leq x \leq a,$$

has length $14/3$.



Question 5 – Solution**Step 1: Differentiate.**

$$y' = \sqrt{x}.$$

Step 2: Write and evaluate the length.

$$\begin{aligned} L &= \int_0^a \sqrt{1+x} \, dx \\ &= \left[\frac{2}{3} (1+x)^{3/2} \right]_0^a \\ &= \frac{2}{3} \left((1+a)^{3/2} - 1 \right). \end{aligned}$$

Step 3: Set the length equal to 14/3 and solve.

$$\begin{aligned} \frac{2}{3} \left((1+a)^{3/2} - 1 \right) &= \frac{14}{3}, \\ (1+a)^{3/2} - 1 &= 7, \\ (1+a)^{3/2} &= 8, \\ 1+a &= 8^{2/3} = 4, \\ a &= 3. \end{aligned}$$

$$\boxed{a = 3}$$